90. (First problem in **Cluster 2**)

The setup for this cluster refers to Fig. 10-16 in the chapter that assumes both angles are positive (at least, this is what is assumed in writing down Eq. 10-43) regardless of whether they are measured clockwise or counterclockwise. In this solution, we adopt that same convention.

(a) We first examine conservation of the y components of momentum:

$$\begin{aligned} 0 &= -m_1\, v_{1f}\, \sin\theta_1 + m_2\, v_{2f}\, \sin\theta_2 \\ 0 &= -m_1(5.00 \text{ m/s}) \sin 30^\circ + (2m_1)\, v_{2f}\, \sin\theta_2 \end{aligned}$$

Next, we examine conservation of the x components of momentum.

$$\begin{aligned} m_1\, v_{1i} &= m_1\, v_{1f}\, \cos\theta_1 + m_2\, v_{2f}\, \cos\theta_2 \\ m_1(10.0 \text{ m/s}) &= m_1(5.00 \text{ m/s}) \cos 30^\circ + (2m_1)\, v_{2f}\, \cos\theta_2 \end{aligned}$$

From the y equation, we obtain $1.25 = v_{2f}\, \sin\theta_2$ with SI units understood; similarly, the x equation yields $2.83 = v_{2f}\, \cos\theta_2$. Squaring these two relations and adding them leads to

$$1.25^2 + 2.83^2 = v_{2f}^2 \left(\sin^2\theta_2 + \cos^2\theta_2\right)$$

and consequently to $v_{2f} = \sqrt{1.25^2 + 2.83^2} = 3.10$ m/s. Plugging back in to either the x or y equation yields the angle $\theta_2 = 23.8^\circ$.

(b) We compute decrease in total kinetic energy:

$$K_i - K_f = 27.9\, m_1$$

so that the collision is seen to be inelastic. We find that

$$\frac{27.9\, m_1}{\frac{1}{2}\, m_1\, 10^2} = 0.558 \ ,$$

or roughly 56%, of the initial energy has been "lost."